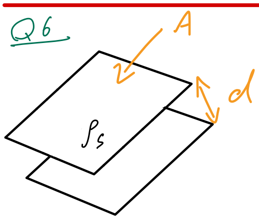
Parallel Plate Capacitor

A, d, ρ_s^{Top}, gap is air, $\rho_s^{bottom} = -\rho_s^{Top}$

Q6



Capacitance

$$\boxed{C = \frac{\varepsilon_0 A[m^2]}{d\,[m]}}$$

$R = \frac{\rho L}{A}$

C in terms of ε_0: $\frac{A}{d}\varepsilon_0$

Voltage

$C = \frac{Q}{V}$ $\quad \therefore \boxed{V = \frac{Q}{C} = \frac{\rho_s A}{C}}$

$Q = \int_s \rho_s \, ds = \rho_s \cdot A$

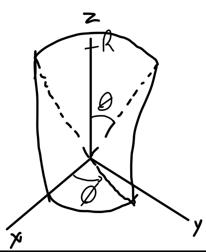
$V = -\int \vec{E} \cdot d\vec{l}$ $\quad \vec{E} = \frac{\vec{D}}{\varepsilon}$ $\quad |\vec{D}| = \rho_s$

Force

$\vec{F} = Q\vec{E}$ $\quad |\vec{E}| = \frac{\rho_s}{2\varepsilon_0}$ $\quad \boxed{|\vec{F}| = Q|\vec{E}|}$

Spherical Dielectric

R, θ, φ, E, ε



Simplify $|E|^2$

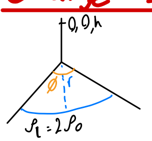
use trig

Electrostatic Energy

$W_e = \frac{1}{2}\varepsilon \int_v |\vec{E}|^2 \underbrace{dv}_{R^2 \sin\theta \, dR \, d\theta \, d\phi}$

Charge Density

ρ_l, r, φ, z,



$\bar{r}$ = point of observation $= h\hat{z}$

$\bar{r}'$ = generic point on source $= a\hat{r}$

$|\bar{r} - \bar{r}'| = \sqrt{a^2 + h^2}$

$dl' = a\,d\phi$

$\vec{E} = \int_l \frac{\rho_l (\bar{r} - \bar{r}')\, dl'}{4\pi\varepsilon_0 |\bar{r} - \bar{r}'|^3}$

$\therefore \vec{E} = \int \frac{2\rho_0 (h\hat{z} - a\hat{r})\, a\, d\phi}{4\pi\varepsilon_0 (\sqrt{a^2+h^2})^3}$

$= \frac{\rho_0}{2\pi\varepsilon_0 (a^2+h^2)^{3/2}} \left[ah\hat{z}\int_0^{\pi/2} d\phi - a^2 \int_0^{\pi/2} \hat{r}\, d\phi \right]$

$\hat{r} = \int_0^{\frac{\pi}{2}} (\cos\phi\,\hat{x} + \sin\phi\,\hat{y})\, d\phi = \hat{x} + \hat{y}$

$$\boxed{\therefore \vec{E} = \frac{\rho}{2\pi\varepsilon(a^2+h^2)^{3/2}} \left[-a^2(\hat{x}+\hat{y}) + \frac{ah\pi}{2}\hat{z} \right]}$$

Charge Density

$Q = \int_0^x \rho_l \, dl = \int_0^x \rho_l a \, d\phi = \rho_l a x$

Net Charge

$Q_{enclosed} = Q = \rho_l a x$

Field Intensity

$\vec{E} = \frac{Q}{4\pi\varepsilon_0 h^2}$

New φ

$\int_0^{2\pi} \cos\phi\,\hat{x} + \sin\phi\,\hat{y}\, d\phi = 0$

$\therefore E = \rho a h / \varepsilon_0 (a^2+h^2)^{3/2}$

Generic formulas

$\int_v dV \Rightarrow R^2 \sin\theta \, dr \, d\theta \, d\phi$

$\int_s dA \Rightarrow R^2 \sin\theta \, d\theta \, d\phi$

$\int_l dl = R\, d\phi$

Trig:

$\sin^2\theta + \cos^2\theta = 1$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$

$\bar{r}$ = point of observation

$\bar{r}'$ = generic point on a source

Electric Flux

$\rightarrow D = A\hat{r} + B\hat{\phi} + C\hat{z}$ $\quad C/m^2$

Gauss' Law

$\rho_v = \nabla \cdot \vec{D} = \frac{1}{r}\frac{\partial}{\partial r}(rD_r) + \frac{1}{r}\frac{\partial}{\partial \phi}(D_\phi) + \frac{\partial}{\partial z}(D_z)$

$= \frac{1}{r}\frac{\partial}{\partial r}(rA) + \frac{1}{r}\frac{\partial}{\partial \phi}(B) + \frac{\partial}{\partial z}(C)$

Total Charge

$\boxed{Q} = \int_v \rho_v \, dv$ $\quad (r\,d\phi\,dr\,dz)$

Flux

$\boxed{Q} = \int \vec{D} \cdot d\vec{s}$

$\phi_1 = \frac{\pi}{2}$

$\phi_2 = \pi$

$Q = \iint D_r \, r\, d\phi\, dz + \iint D_{\phi_1} dr\, dz + \iint D_{\phi_2} dr\, dz$

$\boxed{Q} = \boxed{Q}$

Potential field

Free space: ε_0, V

Electric field intensity

$\vec{E} = -\nabla V = -\frac{\partial V}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{\phi} - \frac{\partial V}{\partial z}\hat{z}$

Volume charge density

$\rho_v = \nabla \cdot \vec{D} = \nabla \cdot \varepsilon\vec{E} = \varepsilon_0 \nabla \vec{E}$

$= \varepsilon_0 \left[\frac{1}{r}\frac{\partial}{\partial r}(rE_r) + \frac{1}{r}\frac{\partial}{\partial \phi}(E_\phi) + \frac{\partial}{\partial z}(E_z) \right]$

Total charge

$Q = \int \rho_v \, dv$

Work done

$W = Q\Delta V = Q(V_A - V_B)$

Potential Field

V, ε_0

Field intensity

$\vec{E} = -\nabla V$

Energy Stored

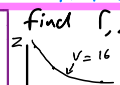
$W_e = \frac{\varepsilon_0}{2}|E|^2$

Charge density

$\rho_v = \nabla \cdot \vec{D} = \nabla \cdot \varepsilon_0 E$

Equipotential Line

find r, z



Work

$W = Q(V_A - V_B)$

Total charge

$Q = \int \rho_v \, dv$

Volume Charge density

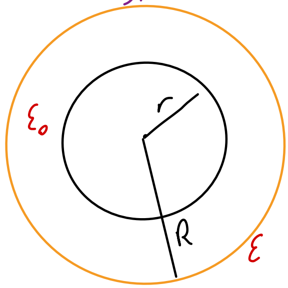
ρ_v, r, ε, R, θ, ϕ

Gauss' Law

$$\oint_S \vec{D}\, d\vec{s} = Q$$

exposed surface

enclosed charge



$$D_r 4\pi R^2 = \rho_v \left(\frac{4}{3}\pi R^3\right)$$

$$D = \frac{\rho_v R}{3R^2}\hat{R}$$

$$D_R = \rho_v \frac{8}{3}$$

$$D = \frac{8\rho_v}{3R^2}\hat{R}$$

Electric field intensity

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{\rho_v R}{3\varepsilon_0 R^2}\hat{R} \qquad R < 2$$

$$\vec{E} = \frac{8\rho_v}{3\varepsilon_0 R^2}\hat{R} \qquad 2 < R < 4$$

$$\vec{E} = \frac{8\rho_v \hat{R}}{3 \cdot 2\varepsilon_0 R^2} = \frac{4\rho_v}{3\varepsilon_0 R^2}\hat{R} \qquad R > 4$$

Electric Potential

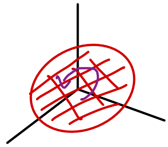
$$V = -\int_R^\infty \vec{E}\, dL = \int_R^\infty \frac{4\rho_v}{3\varepsilon R^2}\, dR = \frac{4\rho_v}{3\varepsilon_0}\left[\frac{1}{R}\right]_R^\infty = \frac{4\rho_v}{3\varepsilon_0 R}$$

Polarization Vector

$$\vec{P} = \chi_e \varepsilon_0 \vec{E} = (\varepsilon_r - 1)\varepsilon_0 \vec{E} = (\varepsilon - \varepsilon_0)\vec{E}$$

Surface Charge Density

ρ_s, r, ϕ, z



$$\vec{r} - \vec{r}' = h\hat{z} - r\hat{r}$$

$$|\vec{r} - \vec{r}'| = \sqrt{r^2 + h^2}$$

$$ds' = dr(r\, d\phi)$$

Flux

$$\Psi = \frac{Q}{2}$$

Net charge

$$\Psi = Q \qquad \text{if } a < 4$$

$$\Psi = \rho_s \pi (4)^2 = 16\pi\rho_s \qquad \text{if } a > 4$$